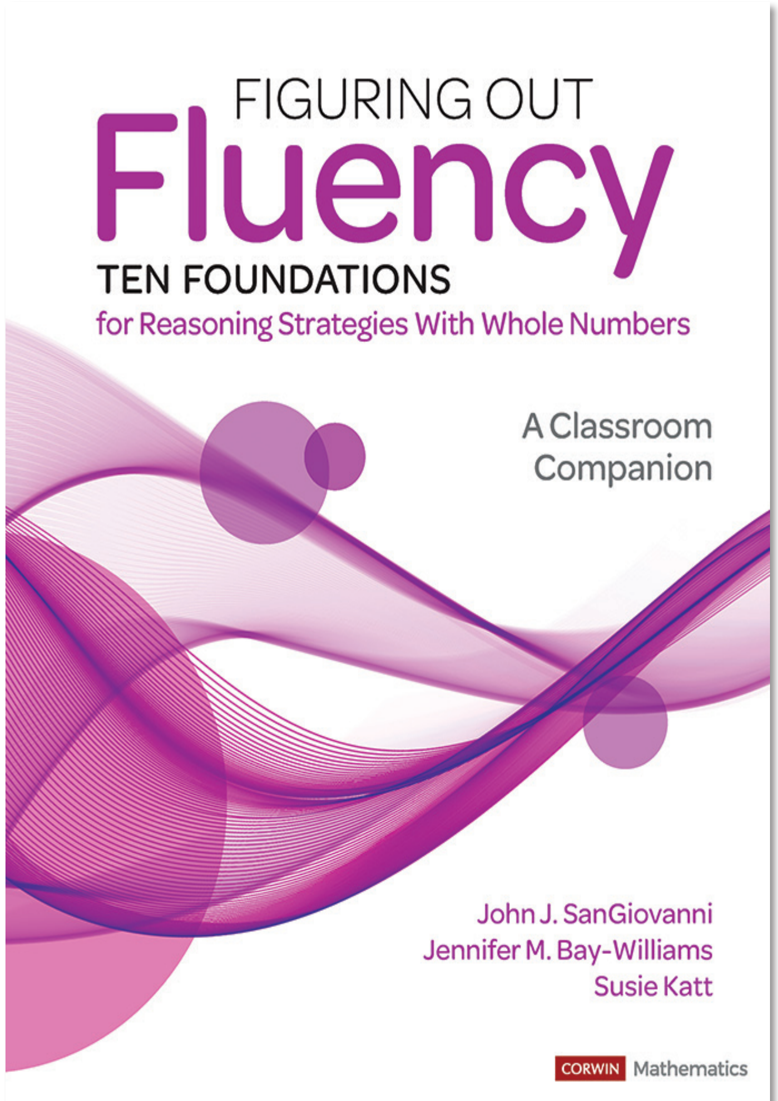

Thank you

FOR YOUR INTEREST IN CORWIN

Please enjoy this complimentary excerpt from Figuring Out Fluency- Ten Foundations for Reasoning Strategies With Whole Numbers.

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CORWIN

WHAT IS FLUENCY IN MATHEMATICS AND WHY IS IT IMPORTANT?

In construction, a foundation is the load-bearing part of a building. So it is with fluency—fluency is built upon foundational concepts and skills. Without such foundations, students are unable to build fluency with basic facts, whole numbers, and more. What makes a strong foundation is based on what one is trying to build. So, we start with the goal of procedural fluency. Try out these problems using any strategy that you like:

$$398 + 535$$

$$504 - 495$$

$$1{,}435 \div 7$$

How did you find the sum of the first example, the difference in the second, and the quotient in the third? Did you use strategies or algorithms? Did you start with one strategy and shift to another? Each of these problems can be solved efficiently using a strategy other than the standard algorithms. For example, in 1,435 ÷ 7, the dividend can be broken apart into 1,400 + 35, and each part can be divided by 7, resulting in 200 + 5. A person who demonstrates fluency with division notices the following:

- The dividend (1,435) includes multiples of 7.

  Foundation: Knowing multiples (Module 8)

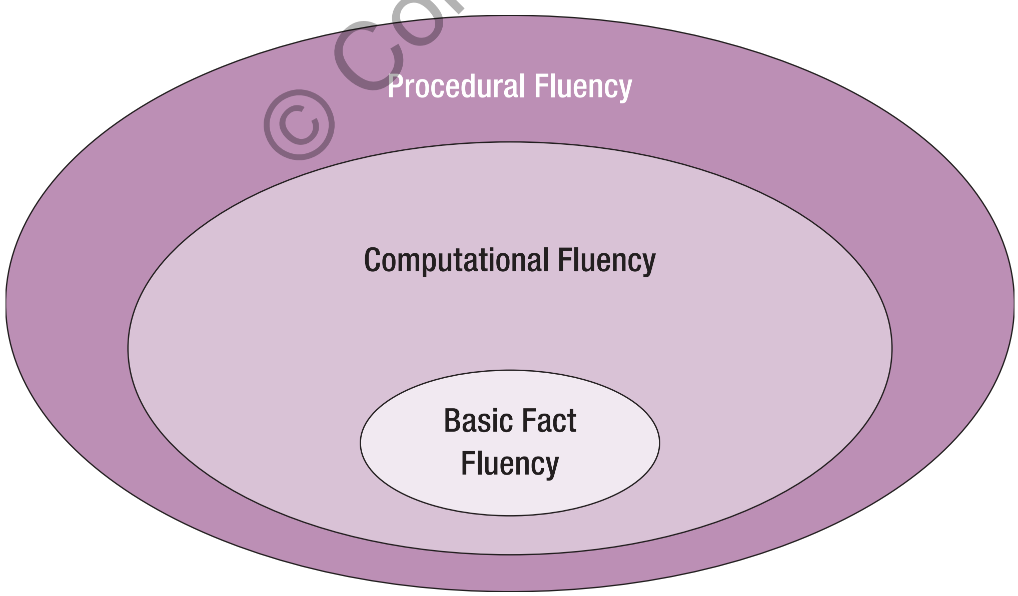
- The dividend can be decomposed into more noticeable multiples of 7 (1,400 + 35; not decomposed by place value).

  Foundation: Being able to flexibly decompose (Module 2)

- If 14 ÷ 7 = 2, then 1,400 ÷ 7 = 200.

  Foundation: Multiplying by tens and hundreds (Module 7)

FIGURE 1 ● The Relationship of Different Fluency Terms in Mathematics



Importantly, with these foundational concepts and skills in place, a person has access to a strategy that is more efficient and less error-prone than long division. Thus, these foundations are the necessary and good beginnings of fluency! Revisit the other problems posed above and ask yourself, "What foundational concepts and/or skills allow me to solve this problem more efficiently than using a standard algorithm?"

Procedural fluency is an umbrella term that includes basic fact fluency and computational fluency (see Figure 1).

Basic fact fluency attends to fluently adding, subtracting, multiplying, and dividing single-digit numbers (see Figure 2).

Computational fluency refers to the fluency in four operations across number types (whole numbers, fractions, etc.), regardless of the magnitude of the number. Procedural fluency encompasses both basic fact fluency and computational fluency plus other procedures, such as finding equivalent fractions.

Procedural fluency is defined as solving procedures efficiently, flexibly, and accurately (National Council of Teachers of Mathematics [NCTM], 2014; National Research Council, 2001). The meaning of these three components are

> *Efficiency:* Solving a procedure in a reasonable amount of time by selecting an appropriate strategy and readily implementing that strategy.
>
> *Flexibility:* Knowing multiple procedures and applying or adapting strategies to solve procedural problems (Baroody & Dowker, 2003; Star, 2005).
>
> *Accuracy:* Correctly solving a procedure.

To focus on fluency, we need specific, observable actions that we can look for in order to assess what students are doing as they solve computational problems. We have identified six such actions. The three components and six fluency actions (and their relationships) are illustrated in Figure 3.

FIGURE 2 ● Basic Fact Strategies and Their Extensions

BASIC FACT STRATEGY	BASIC FACT (SINGLE DIGIT) EXAMPLE	EXTENSIONS TO OTHER TYPES OF NUMBERS
Making 10	7 + 9 = 6 + 10 = 16	97 + 35 = 100 + 32 3.9 + 1.4 = 4 + 1.3
Pretend-a-10 (Compensation)	9 + 6 → 10 + 6 → 16 16 - 1 = 15	3,499 + 5,148 → 3,500 + 5,148 – 1
ThinkAddition	11 - 7 → 7 + ? = 11	89 - 75 → 75 + ? = 89 $9\frac{1}{8} - 8\frac{1}{2} \rightarrow 8\frac{1}{2} + ? = 9\frac{1}{8}$
Doubling	4 × 7 = 2 × 7 × 2	$4 \times 2\frac{1}{2} = 2 \times 2\frac{1}{2} \times 2$ 5 × 28 = 5 × 2 + 14
Add-a-Group	6 × 7 = 5 × 7 + 7	26 × 4 = 25 × 4 × 4
Subtract-a-Group	9 × 8 = 10 × 8 - 8	99 × 8 = 100 × 8 - 8
Think Multiplication	45 ÷ 9 → 9 × ? = 45	14.35 ÷ 7 → 7 × ? = 14.35

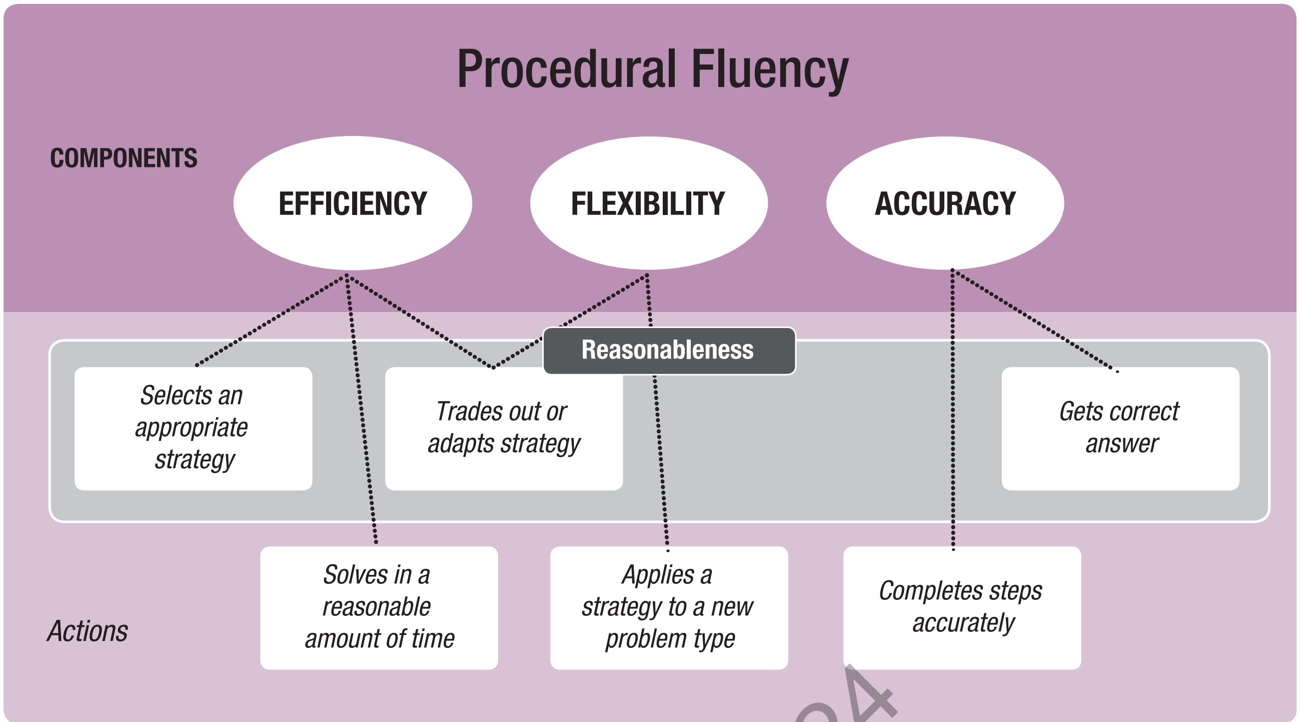
Three of the six fluency actions (should) attend to reasonableness. Fluency actions and reasonableness are described later in Part 1, but first, it is important to consider why this bigger (comprehensive) view of fluency matters. Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy.

Real fluency is not the act of replicating someone else's steps or procedures for doing mathematics. It is the act of thinking, reasoning, and doing mathematics on one's own. The NCTM (2023) Procedural Fluency Position Statement describes what procedural fluency is and what is necessary to ensure all students develop procedural fluency, citing significant research along with instructional resources for classroom support.

TEACHING TAKEAWAY

Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy.

FIGURE 3 ● Procedural Fluency Components, Actions, and Checks for Reasonableness



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