STANDARD 4 (5.NF.B.4)

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. *Interpret the product* $\frac{a}{b} \times q$ *as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations* $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)

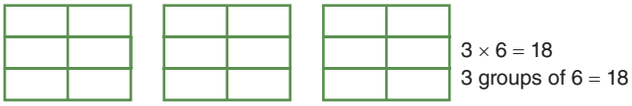
In Grade 4, students used models to multiply a fraction by a whole number (for example, $4 \times \frac{3}{5}$), connecting to the meaning of whole number multiplication. Fifth grade students extend this concept by using models to represent situations in which they need to multiply a whole number by a fraction ($\frac{3}{5} \times 4$) or a fraction by a fraction ($\frac{1}{4} \times \frac{3}{5}$). Provide students with real-life contexts and situations to model in order to give them experiences they need to develop understanding of what is happening when they multiply a fraction by a fraction.

What the TEACHER does:

- Explicitly connect multiplication of whole numbers to multiplication with fractions by giving students connected situations that they can model.
- Scaffold problems beginning with unit fraction factors and build to multiplying with other fractions and mixed numbers.

Example 1:

Frank baked 3 pans of brownies. He cut 6 brownies in each pan. How many brownies did Frank bake? (3 groups of 6)

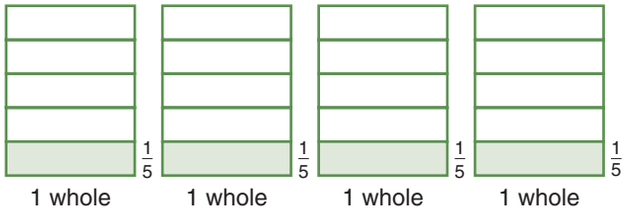


Example 2:

Marcella made 4 gallons of punch. One-fifth of the punch was orange juice. How much orange juice did she use in the punch?

A student might think of this as $\frac{1}{5}$ of each gallon being orange juice. Because there are 4 gallons that would show that $\frac{4}{5}$ of a gallon is orange juice.

$\frac{1}{5} \times 4$



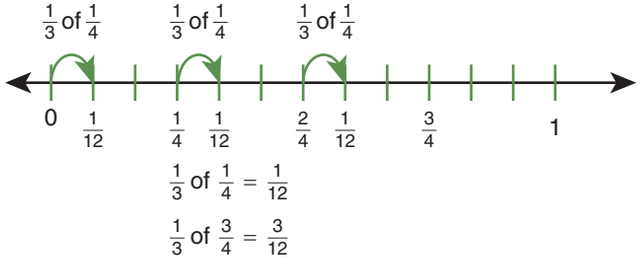
$\frac{1}{5} \times 4$ means $\frac{1}{5}$ of each group of 4 $= \frac{4}{5}$

Example 3:

The distance from Elsa's house to her grandmothers is $\frac{3}{4}$ of a mile. She biked $\frac{1}{3}$ of the way there and stopped to rest. How far did Elsa travel before her rest stop?

A student might say, because I wanted one-third of the distance, I divided each fourth into 3 sections since I thought $\frac{1}{3}$ of the way would be the same as $\frac{1}{3}$ of each fourth. When I put them together, the total distance she biked before resting would be $\frac{3}{12}$ of a mile.

$\frac{1}{3} \times \frac{3}{4}$



- Give students time to work in groups to explore solutions using models, including area models, fraction bars, and number lines.
- Monitor group work by noting what students are doing and asking supporting questions.
- Facilitate class discussions in which students model and explain their thinking.

(continued)

What the TEACHER does (continued):

- Use formative assessment tasks to determine whether students understand what is happening when they multiply two fractions and why the product is smaller than the factors. Students need to understand that because each factor represents a part of the whole, when they multiply a fraction times a fraction, they are taking part of a part. For example, given the example $\frac{1}{2} \times \frac{1}{4}$, students should understand that they are taking a part ($\frac{1}{2}$) of a part ($\frac{1}{4}$) and the result ($\frac{1}{8}$) will be smaller than the part they had at the beginning.
- As students demonstrate conceptual understanding of what happens when they multiply fractions using models, pictures, words, and numbers, encourage students to look for patterns so they can generalize a procedure for multiplying fractions and justify why that procedure works. Why can you multiply numerators and multiply denominators to get the product of two fractions?

What the STUDENTS do:

- Explore what happens when multiplying a whole number by a fraction by solving a variety of word problem contexts using models, pictures, words, and numbers.
- Explain their reasoning to partners, groups, and to the class. Compare different strategies focusing on how strategies are similar and how they are different.
- Explore multiplication of a fraction by a fraction by solving a variety of word problem contexts using models, pictures, words, and numbers.
- Explain their reasoning to partners, groups, and to the class. Compare different strategies focusing on how strategies are similar and how they are different.
- Look for patterns when multiplying fractions. Explain why those patterns work using models, pictures, words, and numbers.
- Apply the patterns to determine a procedure for multiplying fractions.

Addressing Student Misconceptions and Common Errors

Students may see the pattern and see that to multiply fractions you "simply" multiply the numerators and multiply the denominators. This is the correct algorithm or procedure. However, only references to real-life situations and using models and visual representations will help students develop a conceptual understanding of what is actually happening when they multiply fractions.

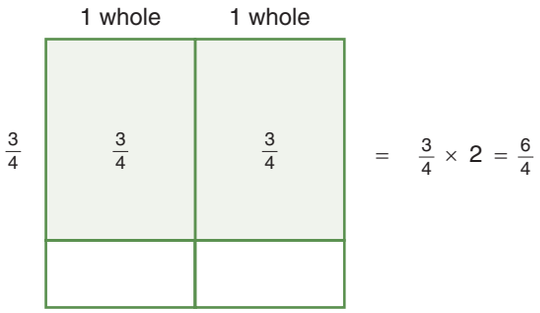
Notes

b. *Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.*

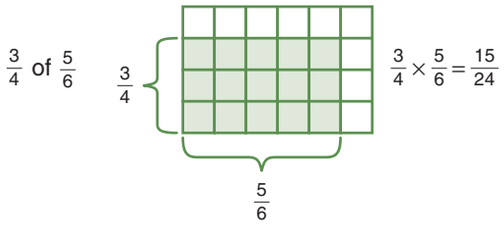
Using area models was a focus of work with multiplication of whole numbers in grades 3 and 4. Fifth graders extend this work to examples with area models that have fractional side lengths. Students should have a variety of problems to solve using area models. This Standard can be taught in conjunction with earlier exploration of multiplication of whole numbers by a fraction, fractions by fractions, and mixed numbers.

What the TEACHER does:

- Provide students with problem contexts in which they find the area of a rectangle with one side that is a fraction and the other side a whole number.
- Facilitate a discussion in which students determine the part of the unit square used to tile the rectangle based on the dimensions of the side.
- Have students work in pairs or groups using grid paper to model the problem and discuss how to find the area of the rectangle.



- Ask students to explain their thinking using pictures, words, and numbers.
- Extend the problem to situations finding the area of a rectangle with both sides as fractions or mixed numbers.
- Have students work in pairs or groups using grid paper to model the problem and discuss how to find the area of the rectangle when one or both sides include a fraction.



- Facilitate classroom discussions in which students explain their reasoning and strategies to solve the problems using pictures, words, and numbers.
- Ask students to compare previous work and generalizations with multiplying fractions and mixed numbers to the solutions of these problems. What is similar? What is different?

What the STUDENTS do:

- Work in groups to solve the area problems using pictures and models.
- Explain the measures of the pieces they use to tile the rectangle, determining what part of a unit square each piece represents.
- Share the strategies and reasoning they use to solve each problem using models, pictures, words, and numbers.
- Compare the process and results of solving these problems with the previous work involving multiplication of fractions.

GRADE 5

Addressing Student Misconceptions and Common Errors

Watch for students who have difficulty determining the part of the unit square. Thinking in terms of the whole rectangle will help them define the number of parts when the dimensions are fractional parts of the whole. Reinforcing when they multiply a fraction by a fraction they are taking part of a part will help students to see that the "overlap" is the number of pieces (or numerator), and the total number of pieces in the whole is the denominator.